

1st Asian-Pacific Summer School on Formal Methods

Course 12: Deductive verification of C programs with Frama-C and Jessie

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CEA List

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Jessie Usage

Function Contracts

Advanced Specification

Example 1: Searching

Example 2: Sorting

long n;
 for (i = 0; i < n; i++)
 C[i] = 0;
 tmp2 = 0;
 // ...

tmp2[0] = 0; for (k = 1; k < n; k++) tmp1[k] += m2[0][k] * tmp2[k]; /* The [j] coefficient of the matrix product MC2*TMP2, that is, *MC2*[TMP1] = MC2*[MC1*M1] = MC2*M1*MC1. l = 1; tmp1[0] += l; */ Final rounding: tmp2[0] is now represented on 9 bits: *if (tmp1[0] < -256) m2[0] = -256; else if (tmp1[0] > 255) m2[0] = 255; else tmp2[0] = tmp1[0];



- ▶ Hoare-logic based plugin, developed at INRIA Saclay.
- ▶ Input: a program and a specification
- ▶ Jessie generates **verification conditions**
- ▶ Use of **Automated Theorem Provers** to discharge the VCs
- ▶ If all VCs are proved, **the program is correct** with respect to the specification
- ▶ Otherwise: need to investigate why the proof fails
 - ▶ Fix bug in the code
 - ▶ Adds additional annotations to help ATP
 - ▶ Interactive Proof (Coq)



Usage

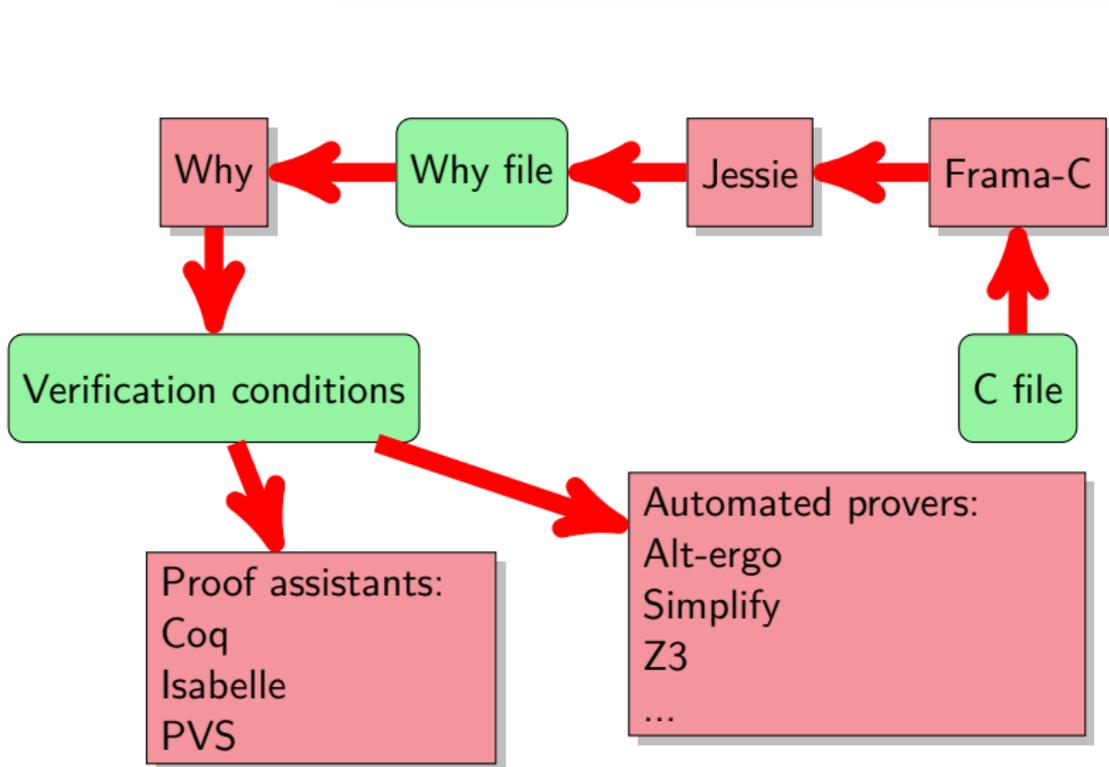
- ▶ Proof of functional properties of the program
- ▶ Modular verification (function per function)

Limitations

- ▶ Cast between pointers and integers
- ▶ Limited support for union type
- ▶ Aliasing requires some care



From Frama-C to Theorem Provers



$$tmp2[0] = 1 \wedge \dots \wedge (n-1) \wedge \dots \wedge tmp[0] = 1 \wedge \dots \wedge (n-1) \wedge \dots \wedge tmp2[0] = tmp[0] \wedge \dots \wedge tmp2[n-1] = tmp[n-1] \wedge \dots$$

$$tmp[0] = 0 \wedge k = 5 \wedge k = tmp[0] \wedge \dots \wedge mc2[0][k] = tmp2[0][k] \wedge \dots$$

$$Final\ condition\ tmp2[0] \text{ is now represented on 3 bits: } tmp[0] < 256 \wedge tmp[0] > 255 \wedge tmp[0] > 255 \wedge tmp[0] < 256 \wedge \dots$$

Check safety of a function

- ▶ Pointer accesses
- ▶ Arithmetic overflow
- ▶ Division

```
unsigned int M;
```

```
void mean(unsigned int* p, unsigned int* q) {
    M = (*p + *q) / 2;
}
```



```
unsigned int M;
```

```
/*@ requires \valid(p) ^ \valid(q);
 */
```

```
void mean(unsigned int* p, unsigned int* q) {
  if (*p ≥ *q) { M = (*p - *q) / 2 + *q; }
  else { M = (*q - *p) / 2 + *p; }
}
```



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```

(long n)
for (i = 0; i < n; i++)
    C[i] = i * i;
tmp2 = ...
// ...

```

```

tmp2[0] = 1; // (N-1) else if (tmp1[0]) >= 1; // (N-1) - 1; else tmp2[0] = tmp1[0]; // Then the second pass looks like the first one:
tmp1[0] = 0; k = 0; k++ tmp1[0][k] += mc2[0][k] * tmp2[k]; // The [i][j] coefficient of the matrix product MC2*TMP2, that is, *MC2*(TMP1) = MC2*(MC1*M1) = MC2*M1*MC1
i = 1; tmp1[0][i] >= 1; // Final rounding: tmp2[0][i] is now represented on 9 bits: *if (tmp1[0][i] < -256/m2[0][i]) = -256; else if (tmp1[0][i] > 255/m2[0][i]) = 255; else tmp2[0][i] = tm

```



Safety of a program is important, but this is not sufficient: We also want it to do “the right thing”...

But in order for jessie to verify that, we need to explain it what “the right thing” is, and to explain it formally

This is the purpose of ACSL, ANSI/ISO C Specification Language.

- ▶ Behavioral specification language à la JML and Eiffel
- ▶ Function contracts
- ▶ Logic models
- ▶ Independent from any plug-in



- ▶ Functional specification
- ▶ Pre-conditions (requires)
- ▶ Post-conditions (ensures)

Example

```

unsigned int M;
/*@
  requires \valid(p) ^ \valid(q);
  ensures M ≡ (*p + *q) / 2;
*/
void mean(unsigned int* p, unsigned int* q) {
  if (*p ≥ *q) { M = (*p - *q) / 2 + *q; }
  else { M = (*q - *p) / 2 + *p; }
}

```



One can define logic functions as follows:

```
/*@ logic integer mean(integer x, integer y) =
    (x+y)/2; */
```

long ra
 for 0 <=
 C1) if (a
 tmp2 =
 re of the

tmp2[0] = 1 << (nbl - 1) else if (tmp1[0]) >= 1 << (nbl - 1) tmp2[0] = (1 << (nbl - 1)) + tmp1[0]; 7. Then the second part takes the first part and
 tmp1[0][k] = 0; k = 8; k++) tmp1[0][k] += mc2[0][k] * tmp2[0][k]; 7. The [i][j] coefficient of the matrix product MC2*TMP2, that is, *MC2*(TMP1) = MC2*(MC1*M1) = MC2*M1 *MC1
 l = 1; tmp1[0][l] >= 1; 7. Final rounding: tmp2[0][l] is now represented on 9 bits. *if (tmp1[0][l] < -256) m2[0][l] = -256; else if (tmp1[0][l] > 255) m2[0][l] = 255; else m2[0][l] = tmp1[0][l];



The specification:

```
/*@
```

```
  requires \valid(p) ^ \valid(q);
```

```
  ensures M ≡ (*p + *q) / 2;
```

```
*/
```

```
void mean(unsigned int* p, unsigned int* q);
```

An admissible implementation:

```
void mean(int *p, int* q) {
```

```
  *p = *q = M = 0; }
```



The specification:

```
/*@
  requires \valid(p) ^ \valid(q);
  ensures M ≡ (*p + *q) / 2;
  ensures *p ≡ \old(*p) ^ *q ≡ \old(*q);
*/
void mean(unsigned int* p, unsigned int* q);
```

An admissible implementation:

```
void mean(int *p, int* q) {
  if (*p ≥ *q) ... else ...
  A = 0; }
```



The specification:

```
/*@
  requires \valid(p) ^ \valid(q);
  ensures M ≡ (*p + *q) / 2;
  assigns M;
*/
```

```
void mean(unsigned int* p, unsigned int* q);
```

An admissible implementation:

```
void mean(int *p, int* q) {
  if (*p ≥ *q) { M = (*p - *q) / 2 + *q; }
  else { M = (*q - *p) / 2 + *p; }
}
```



- ▶ Post condition true *when the function exits normally*.
- ▶ By default, a function always terminates...
- ▶ ... as long as its pre-condition holds.

```

/*@
  requires \valid(p) ^ \valid(q);
  ensures ...
  assigns M;
  terminates \true;
*/
void mean(int* p, int* q) {

  if (p == NULL ∨ q == NULL) while(1);
  if (*p ≥ *q) ...
}
    
```



Jessie Usage

Function Contracts

Advanced Specification

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Example 2: Sorting

```

long n;
for (i = 0; i < n; i++)
    C[i] = 0;
tmp2 = ...
// ...

```

```

tmp2[0] = 1; // C[0] = 1; also if tmp1[0] >= 1 // C[0] = 1; tmp2[0] = 1; // Then the second part looks like the first part.
tmp1[0] = 0; k = 5; k--; tmp1[0] += m2[0][k] * tmp2[k]; // The [i][j] coefficient of the matrix product MC2*TMP2, that is, *MC2*(TMP1) = MC2*(M1)*M1 = MC2*M1*M1
i = 1; tmp1[0] >= 1; // Final rounding: tmp2[0] is now represented on 9 bits. *if (tmp1[0] < -256) m2[0] = -256; else if (tmp1[0] > 255) m2[0] = 255; else m2[0] = tmp1[0];

```



Informal spec

- ▶ Input: a **sorted** array and its length, an element to search.
- ▶ Output: index of the element or -1 if not found

Implementation

```
int find_array(int* arr, int length, int query) {
    int low = 0;
    int high = length - 1;
    while (low ≤ high) {
        int mean = low + (high - low) / 2;
        if (arr[mean] ≡ query) return mean;
        if (arr[mean] < query) low = mean + 1;
        else high = mean - 1;
    }
    return -1;
}
```



What does “sorted” mean?

```
/*@
```

```
  predicate sorted{L}(int* arr, integer length) =  

  ∃ integer i,j; 0 ≤ i ≤ j < length ⇒ arr[i] ≤ arr[j];
```

```
*/
```

ACSL predicates

- ▶ Give a formal definition of the properties of objects
- ▶ Can be used in annotations
- ▶ Must be tied to a program point when performing a memory access
- ▶ Some predicate can relate two or more program points (see after)



find_array has two distinct **behaviors**, depending on whether the query is in the array or not. This can be reflected in the contract in the following way:

```
/*@
```

```
behavior exists:
```

```
assumes  $\exists$  integer  $i$ ;
```

```
 $0 \leq i < \text{length} \wedge \text{arr}[i] \equiv \text{query};$ 
```

```
ensures  $\text{arr}[\backslash\text{result}] \equiv \text{query};$ 
```

```
behavior not_exists:
```

```
assumes  $\forall$  integer  $i$ ;
```

```
 $0 \leq i < \text{length} \implies \text{arr}[i] \not\equiv \text{query};$ 
```

```
ensures  $\backslash\text{result} \equiv -1;$ 
```

```
*/
```



Role of the loop invariant

- ▶ Must be inductive (if it holds at the beginning, then it holds at the end)
- ▶ Capture the effects of one loop step
- ▶ Represents the only things known at the exit of the loop
- ▶ Must be strong enough to allow to derive the post-condition

Example

/*@

```

loop invariant 0 ≤ low;
loop invariant high < length;
loop invariant ∀ integer i;
    0 ≤ i < low ⇒ arr[i] < query;
loop invariant ∀ integer i;
    high < i < length ⇒ arr[i] > query;
  */
  
```

*/



Total correctness

- ▶ Needed for proving termination
- ▶ Expression which strictly decreases at each step
- ▶ And stay non-negative

Example

```
/*@ loop invariant ...
   loop variant high - low;
*/
```



Usage

- ▶ A property which must hold at a given point
- ▶ Allows to guide the automated provers
- ▶ Can be associated to a particular behavior

Example

```
int mean = low + (high - low) / 2;
/*@ assert low ≤ mean ≤ high;
if (arr[mean] ≡ query)
  /*@ for not_exists: assert \false;
```



Informal specification

- ▶ Input: an array and its length
- ▶ Output: the array is sorted in ascending order

```
int index_min(int* a, int low, int high);
```

```
void swap(int* arr, int i, int j);
```

```
void min_sort(int* arr, int length) {
    for(int i = 0; i < length; i++) {
        int min = index_min(arr,i,length);
        swap(arr,i,min);
    }
}
```



- ▶ Case definition: $H_i \Rightarrow Pred$
- ▶ Horn clause: $Pred$ can only appear positively in H_i (ensures consistency)
- ▶ Smallest fixpoint: predicate holds iff one of the H_i holds

```

/*@ inductive Permut{L1,L2}
    (int a[], integer l, integer h) {
    case Permut_trans{L1,L2,L3}:
        ∀ int a[], integer l, h;
        Permut{L1,L2}(a, l, h) ∧
        Permut{L2,L3}(a, l, h) ⇒
            Permut{L1,L3}(a,l, h) ;
    case Permut_swap{L1,L2}: ...
    }
    @*/

```



It is also possible to have axiomatic definitions of predicates and logic functions:

```

/*@ axiomatic Permut{
predicate permut{L1,L2}(int* arr, integer length);
axiom perm_refl{L}:
   $\forall$  int* arr, integer length; permut{L,L}(arr,length);
axiom perm_swap{L1,L2}:
   $\forall$  int* arr, integer length,i,j;
     $0 \leq i < \text{length} \wedge 0 \leq j < \text{length} \implies$ 
    swap{L1,L2}(arr,i,j)  $\implies$  permut{L1,L2}(arr,length);
axiom perm_trans{L1,L2,L3}:  $\forall$  int* arr;
   $\forall$  integer length; permut{L1,L2}(arr,length)  $\implies$ 
  permut{L2,L3}(arr,length)  $\implies$ 
  permut{L1,L3}(arr,length);}*/
  
```

